

On the possible space-time fractality of the emitting source

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February 1, 2008

Abstract

Using simple space-time implementation of the random cascade model we investigate numerically a conjecture made some time ago which was joining the intermittent behaviour of spectra of emitted particles with the possible fractal structure of the emitting source. We demonstrate that such details are seen, as expected, in the Bose-Einstein correlations between identical particles.

PACS numbers: 12.40c 13.78Fh 02.50Ng 15.45+b

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1 Introduction

The multiparticle spectra of secondaries produced in high energy collision processes are the most abundant sources of our knowledge of the dynamics of such processes. Among others, two features emerging from the analysis of these spectra are of particular interest: (i) the so called intermittent behaviour observed in many experiments in the analysis of factorial moments of spectra of produced secondaries and (ii) the Bose-Einstein correlations (BEC) observed between identical particles. Whereas the former seems to indicate the existence of some (multi) fractal structure of the production process [1] the latter are established as, by now the most important source of our knowledge on the space-time aspects of the multiparticle production processes [2].

Some time ago it was argued [3, 4] that, in order to make both effects compatible with each other, the emitting source should fluctuate in size in a scale-invariant (i.e., power-like) way. This can be achieved in two ways: (i) either the shape of the interaction region is regular but its size fluctuates from event to event according to some power-like scaling law (ii) or the interaction region itself is a self-similar fractal extending over a very large volume [3, 5].

In this work we would like to investigate in more detail to what extent the BEC is sensitive to the possible space-time fractality of the emission source. To this end we shall use a simple self-similar cascade process [6] in which the final particles are produced in the sequential two-body decays of some original mass M . For our purpose we shall extend it by introducing the simple (classical) space-time development of the cascade and by adding the kind of BEC “afterburner” along the lines advocated recently in [7].

It is widely expected that every cascade model has automatically built in the intermittent behaviour of spectra of observed particles [8]. Although this statement is true and obvious for the models based on random multiplicative processes in some chosen observed variables (like energy, rapidity or azimuthal angle) it is highly non trivial in the case of cascades (or multiplicative processes) proceeding in variable(s) not directly measurable but nevertheless of great dynamical importance (as, for example, masses of some intermediate objects occurring during the production process [6]). In the purely mathematical case, where cascade process proceeds *ad infinitum*, one eventually arrives at some space-time fractal picture of the production process. However, both the finite masses of produced secondaries and limited energy (or

mass M) stored in the emitting source prevent the full and distinct development of such a fractal structure [9]. One must therefore be satisfied with only some limited and mostly indirect presence or signals of such structure. If established it would, however, be important for our knowledge of the dynamics of the multiparticle production process.

Such a fractal structure in phase-space can generate a similar structure in the space-time picture of the hadronization process. Our aim here is to demonstrate to what extent they influence BEC. In the next two Sections we shall then provide, respectively, the phase-space and space-time characteristics of a simple cascade model used for that purpose. Section 4 contains our main results showing the BEC features emerging from our model. Section 4 contains a summary of our results and conclusions.

2 Phase-space characteristic of the cascade model used

We shall model the emitting source of mass M by the usual $(1 \rightarrow 2)$ random cascade process employed already in [6], $M \longrightarrow M_1 + M_2$, in which the initial mass M “decays” into two masses $M_{1,2} = k_{1,2} \cdot M$ in such a way that $k_1 + k_2 < 1$, i.e., a part of M equal to $(1 - k_1 - k_2)M$ is transformed to kinetic energies of the decay products $M_{1,2}$. The process repeats itself (see Fig. 1) until $M_{1,2} \geq \mu$ (μ being the mass of the produced particles) with successive branchings occurring sequentially and independently from each other, and with different values of $k_{1,2}$ at each branching, but with energy-momentum conservation imposed at each step. For different choices of dimensionality D of our cascade process (provided by the restrictions for the possible directions of flights of the decay products in each vertex) be it $D = 1$ or $D = 3$ dimensional (isotropic) and for different choices of the decay parameters $k_{1,2}$ at each vertex, we are essentially covering an enormously vast variety of different possible production schemes ranging from the essentially one-dimensional strings to thermal-like fireballs.

Of special interest is the case of a one-dimensional cascade for which one can provide analytic formulae for the rapidities $Y_{1,2}$ of the decay product at each vertex given in the rest frame of the parent mass in this vertex. They depend solely on the

decay parameters at this vertex, $k_{1,2}$:

$$\begin{aligned} Y_1 &= \pm \ln \left[\frac{1}{2k_1} (1 + k_1^2 - k_2^2) + \frac{1}{2k_1} \sqrt{\Delta} \right], \\ Y_2 &= \mp \ln \left[\frac{1}{2k_2} (1 - k_1^2 + k_2^2) + \frac{1}{2k_2} \sqrt{\Delta} \right], \\ \text{where } \Delta &= (1 - k_1^2 + k_2^2)^2 - 4k_2^2. \end{aligned} \quad (1)$$

Two limiting cases can be distinguished here: (i) - totally symmetric and (ii) maximally asymmetric cascades. In the case of a totally symmetric cascade decay parameters are equal and the same for all vertices, $k_{1,2} = k$. In this case the finally produced particles occur only at the very end of the cascade process and the amount of energy allocated to the production is maximal. Because the number of possible branchings characterizing the length of the cascade is equal to $L_{max} = \ln \frac{M}{\mu} / \ln \frac{1}{k}$ (where $\mu = \sqrt{m_0^2 + \langle p_T \rangle^2}$), the multiplicity of produced secondaries is given by the following formula:

$$N_s = 2^{L_{max}} = \left(\frac{M}{\mu} \right)^{d_F}, \quad d_F = \frac{\ln 2}{\ln \frac{1}{k}}. \quad (2)$$

According to [6, 10] the exponent d_F is formally nothing but a generalized (fractal) dimension of the fractal structure of phase space formed by our cascade. The utility of such notion is, however, greatly reduced because of the necessary limited length of our cascades [9]. Notice a kind of scaling in (2) where, for a fixed ratio $\frac{M}{\mu}$ the observed multiplicity N_s depends solely on the decay parameter k . The characteristic power-like behaviour of $N_s(M)$ in (2) is normally attributed to thermal models. For example, for $k = \frac{1}{4}$ one has $N_s \sim M^{\frac{1}{2}}$, which in thermal models would correspond to the ideal gas equation of state with velocity of sound $c_0 = \frac{1}{\sqrt{3}}$ [11]. The same behaviour (on average) is obtained for $k_{1,2}$ chosen randomly from a triangle distribution $P(k) = (1 - k)^a$ with $a \simeq 1$ which will be therefore used in all our numerical calculations. The fact that $N_s \in \left(2, \frac{M}{\mu} \right)$ means that decay parameters are limited to $k \in \left(\frac{\mu}{M}, 1 \right)$.

For maximally asymmetric cascades $k_1 = \frac{\mu}{M}$ and $k_2 = k$, i.e., at each step one has always a single final particle of transverse mass μ produced against some recoil mass $M_1 = kM$: $M \rightarrow \mu + M_1$. The amount of kinetic energy allocated to the produced secondaries is now maximal. The corresponding rapidities Y_1 and Y_2 are now given by eqs. (1) with, respectively, $k_1 = \frac{\mu}{M}$ and $k_2 = k$. Also here the resultant multiplicity N_a is given by the length L_{max} of the cascade, which is limited by the condition $M_l = k^l M \geq \mu$. It means therefore that $L_{max} = \ln \frac{M}{\mu} / \ln \frac{1}{k}$ and the

corresponding multiplicity is

$$N_a = 1 + L_{max} = 1 + \frac{1}{\ln \frac{1}{k}} \cdot \ln \frac{M}{\mu}. \quad (3)$$

The energy dependence of N_a is now logarithmic (in thermal models it would correspond to a one dimensional fireball with $c_0 \rightarrow 1$ [12]) but the same kind of scaling as in eq. (2) is present also here.

Our simple model therefore covers all possible energy dependencies of the multiplicities of produced particles which depend solely on decay parameters $k_{1,2}$ of the cascade and scale in the ratio $\frac{M}{\mu}$. This remains true for both one and three dimensional cascades.

In Fig. 2 one can see examples of rapidity distributions $\frac{1}{N} \frac{dN}{dy}$ calculated for symmetric and asymmetric $D = 1$ cascades discussed above. They are compared there with most probable distributions in one-dimension obtained by means of information theory arguments [13]

$$f_{IT}(y) = \frac{1}{Z} \cdot \exp[-\beta \cdot \mu \cosh y], \quad (4)$$

where $\int dy f(y) = 1$ (what defines Z) and $\beta = \beta(M, N)$ is the corresponding lagrange multiplier ensuring proper conservation of energy-momentum in the case when N particles, each of transverse mass μ , are produced from the source of mass M . Contrary to the case of production via the cascade process, nothing is now said about the production mechanism. It is just tacitly assumed that all produced particles occur, in a sense, instantaneously in the whole allowed phase space with the weights provided by eq. (4), which was obtained by the maximalization of the suitably defined information entropy corresponding to the production process under consideration [13]. It occurs that for a wide range of M and N the quantity $\bar{\beta} = \beta \frac{M}{N}$ is (almost) constant as a function of energy per particle $\bar{m} = \frac{M}{N}$, i.e., also here one encounters a kind of scaling, namely that $\frac{1}{N} \frac{dN}{dy} \sim F\left(z = \frac{\mu \cosh y}{\bar{m}}\right)$.

The shape of the multiplicity distribution, $P(N)$, in our case of a source with fixed ratio $\frac{M}{\mu}$ is given by distribution $P(k_{1,2})$ of decay parameters $k_{1,2}$. We shall use a simple triangle form for it (as already mentioned above) $P(k) = (1-k)^a$, which for $a \simeq 1$ provides the commonly accepted energy behaviour of the mean multiplicities $N(M) \sim M^{0.4 \div 0.5}$ as discussed above. The example of $P(N)$ for $D = 1$ cascades are shown in Fig. 3 ($P(N)$ for $D = 3$ cascades are the same). They exemplify three different choices of the ratio $\frac{M}{\mu}$ ($\frac{10}{0.3} = 33.3$, $\frac{40}{0.3} = 133.3$ and $\frac{100}{0.3} = 333.3$, respectively).

In Fig. 4, we show the behaviour of scaled (“horizontal” [14]) factorial moments F_l (for $l = 2, 3$) calculated for a one-dimensional cascade in rapidity space as a function of number of bins $n_{bin} = Y/\delta y$ (where Y is taken as the corresponding rapidity range for the corresponding mass M and δy denotes the bin size considered):

$$F_l = n_{bin}^{l-1} \left\langle \sum_{i=1}^{n_{bin}} \frac{n_i (n_i - 1) \cdots (n_i - l + 1)}{\langle N \rangle^l} \right\rangle. \quad (5)$$

The N produced particles are distributed among n_{bin} bins with n_i particles in the i^{th} bin, i.e., $\sum_{i=1}^{n_{bin}} n_i = N$. The average is over all events. It is interesting to note that our results, although obtained for essentially the same type of cascade as discussed in [6], apparently demonstrate much stronger intermittency signal than the experimental one shown there. However, no fit to the data was attempted in our case as we are concerned with the properties of a single elementary source only (leaving the problem of their distribution in mass $P(\frac{M}{\mu})$ aside). On the other hand, moments F_l in [6] were in reality not calculated but deduced from experimental data by means of some simple formula obtained from general (mathematical) fractal analysis of symmetric cascade processes. The aim was to deduce from them the fractal dimensions $d_F = \ln 2 / \ln \frac{1}{k}$ of cascade process considered. As a result the corresponding decay parameter in [6] turns out to be very large, $k \simeq 0.45$, leading to $\langle N(M) \rangle \simeq M^{0.8 \div 0.9}$ instead of expected $\langle N(M) \rangle \simeq M^{0.4 \div 0.5}$ as discussed above [11].

3 Space-time characteristic of the cascade model used

We shall now endow our cascade in phase-space with space-time elements (not addressed in [6]). To this aim we introduce some fictitious finite “life time” t for each vertex mass M_l , which is allowed to fluctuate according to some prescribed distribution law $\Gamma(t)$. This procedure is a purely classical one, i.e., we are not treating M_l as resonances, as was done, for example, in [16] on another occasion. Instead, they are regarded to be real particles with masses given by the corresponding values of decay parameters $k_{1,2}$ and with the respective velocities equal to $\vec{\beta} = \frac{\vec{P}_{1,2}}{E_{1,2}}$ ($(E_{1,2}; \vec{P}_{1,2})$ are the energy-momenta of the corresponding decay product given in the rest frame of the parent mass in each vertex). The energy-momentum and charges are strictly conserved in each vertex separately (this is another difference with the information theory approach [13] where such conservation laws are imposed on the whole process instead). As for a decay/branching law we shall choose it in the simplest possible

exponential form:

$$\Gamma(t) = \frac{1}{\tau} \cdot \exp \left[-\frac{t}{\tau} \right] \quad (6)$$

It is straightforward to get our cascade model in the form of a Monte Carlo code. The main features of a one-dimensional case has already been demonstrated above. The only difference between one and three-dimensional cascades is in the fact that, whereas in the former decay products can flow only along one, chosen direction, in the latter in each vertex the flow direction is chosen randomly from the isotropic angular distribution. To allow for some nonzero transverse momentum in the one-dimensional case we are using the transverse mass $\mu = 0.3$ GeV. For the three-dimensional cascade μ is instead set simply to the pion mass, $\mu = 0.14$ GeV. In every case all decays are described in the rest frame of the corresponding parent mass in a given vertex. To get the final distributions, one has to perform a number of Lorentz transformations to the rest frame of the initial source mass M . As output we are getting in each run (event) a number N_j of secondaries of mass μ with both defined energy-momenta $\left[E_j = \sqrt{\mu^2 + \vec{P}_j^2}; \vec{P}_j \right]_{i=1, \dots, N_j}$ and space-time coordinates $[t_j; \vec{r}_j]_{i=1, \dots, N_j}$ of the last branching (i.e., the coordinates of birth of each particle).

Fig. 5 shows densities $\rho(r)$ of points of production for all cases investigated here: for $D = 1$ and 3 dimensional cascades with both constant and mass dependent evolution parameter τ and for three choices of the source mass, $M = 10, 40$ and 100 GeV. As one can see the widely expected (cf. [3, 5]) power-like behaviour of cascading source

$$\rho(r) \sim \left(\frac{1}{r} \right)^L \quad r > r_0, \quad (7)$$

is seen only (if at all) for $r > r_0$, i.e., for radii larger then some (not sharply defined) radius r_0 , value of which depends on all parameters present here: mass M of the source, dimensionality D and evolution parameter τ of the cascade. Below r_0 the $\rho(r)$ is considerably bended, remaining even almost flat for $D = 1$ cascades. Only for rapidly developing cascades (i.e., for $\tau \sim \frac{1}{M}$) in $D = 3$ dimensions, the expected scaling sets in almost from the very beginning and it practically does not depend on the mass of the source. For the limiting case of $M = 100$ GeV the corresponding values of parameter L vary from $L = 1.89$ and $L = 1.86$ for $\tau = 0.2$ and $1/M$ for one dimensional cascades to $L = 2.78$ and $L = 2.8$ for three dimensional cascades.

The shapes of $\rho(r)$ scale in the ratio $\left(\frac{M}{\mu} \right)$ in the same way as the multiplicity distributions $P(N)$ discussed before. As desired power-like behaviour [3]-[5] sets

in (at least approximately) only for long cascades (large values of $\left(\frac{M}{\mu}\right)$) and/or for fast ones (small values of τ), it remains therefore to be checked whether (and to what extent) such conditions are indeed met in the usual hadronic processes. This point is, however, outside of the scope of the present work.

4 Cascades and BEC

Let us proceed now to our main point, namely to the question of whether one can see in BEC some special features which could be attributed solely to the the branchings and to their space-time and momentum space structure. At first glance, the answer seems to be plainly negative as it is easy to check that the function

$$C_2(Q = |p_i - p_j|) = \frac{dN(p_i, p_j)}{dN(p_i) dN(p_j)} \quad (8)$$

does not show any structure of BEC type. It is also true if we endow our cascade process with the production of charges of the type: $\{0\} \rightarrow \{+\} + \{-\}$, $\{+\} \rightarrow \{+\} + \{0\}$ and $\{-\} \rightarrow \{0\} + \{-\}$. In this case C_2 calculated for like charge pairs also does not show any correlations. That is, however, to be expected, because the only way to have (8) showing “primordial” BEC is to introduce them from the very beginning, for example in the way done recently in [15]. Using the same information theory approach as in [13] but adding a new piece of information, namely that the produced particles are mostly bosons and as such they should be grouped together as much as possible in the phase space cells, the authors of [15] indeed obtained a substantial BEC signal, namely $C_2 > 1$ for $p_i \rightarrow p_j$ in (8). No space-time structure was discussed in [15], however.

We cannot follow this prescription here without invoking a kind of extremely difficult to formulate (or calculate) special final state interactions between produced secondaries. Our cascade is supposed to mimic the production process in its development, whereas the information theory procedure of [15] makes no statements whatsoever about the development of the proces as such. It only provides the least biased and at the same time most probable distributions, limited only by imposed constraints of the energy-momentum and (mean) number of particles conservation (reflection of which are the two lagrange multipliers, $\beta(M, N)$ and $\mu(M, N)$, representing in terminology of the usual thermal models the “inverse temperature” and “chemical potential”, respectively. We could, in principle, use the pseudopotential method as, for example, advocated long ago in [17], but this causes changes in the

particle distributions and/or destroys the energy-momentum balance which has to be later restored in a more or less *ad hoc* way and it does not use the information on the space-time structure of our results.

An open question is the possible existence of phase factors in every branch point, which would endow each particular branch and through it also the finally produced secondaries with some specific, possibly path dependent phases. This question is, however, left open here and it is understood that they are all set equal to unity. They would be important to BEC correlations influencing especially values of $C_2(Q=0)$, i.e., the so called degree of coherence/chaoticity λ . We shall return to this problem elsewhere.

Because our aim is not data fitting but checking if, and to what extent, the BEC via its C_2 observable is sensitive to different choices of the cascade processes provided by different sets of parameters, we have decided to use the ideas of the BEC “afterburners” advocated recently in [7]. And because we are not so much interested in particular values of the “radius” and “coherence” parameters R and λ , but in the systematics emerging from our study, we shall use for this exploratory research the most primitive, classical version of such “afterburner”. The procedure we use is therefore very simple. After generating a set of $i = 1, \dots, N_l$ particles for the l^{th} event we choose all pairs of the same sign and endow them with the weight factors of the form

$$C = 1 + \cos [(r_i - r_j)(p_i - p_j)] \quad (9)$$

where $r_i = (t_i, \vec{r}_i)$ and $p_i = (E_i, \vec{p}_i)$ for a given particle.

The results obtained from $N_{event} = 50000$ events are presented in Fig. 6. They are displayed for the same sequence of parameters M (mass of the source), D (its dimensionality) and τ (the evolution parameter) as in Fig. 5. The characteristic feature to be noted is a substantial difference between $D = 1$ and $D = 3$ dimensional cascades both in the widths of the $C_2(Q)$ and their shapes. Whereas the former are more exponential-like the latter are more gaussian-like with a noticeably tendency to flattening out at very small values of Q . Also values of intercepts, $C_2(Q=0)$, are noticeable lower for $D = 3$ cascades. There is also a difference between “slow” (constant τ) and “fast” ($\tau \sim \frac{1}{M}$) cascades, especially for $D = 1$ ones. The former lead to substantially different shapes in this case. In $D = 3$ this effect is not so visible, although it is also present. The length of the cascade (i.e., the radius of the production region, cf. discussion of density ρ before) dictates the width of $C_2(Q)$. However,

the $\left(\frac{M}{\mu}\right)$ scaling observed before in multiplicity distributions and in shapes of source functions is lost here. This is because C_2 depends on the differences of the momenta $p = \mu \cosh y$, which do not scale in $\frac{M}{\mu}$. The flattening mentioned above together with $C_2(0) < 2$ for $D = 3$ cascades are the most distinctive signature of the fractal structure combined with $D = 3$ dimensionality of the cascade. The correlations of the position-momentum type existing here as in all flow phenomena are, in the case of $D = 3$ cascades, not necessarily vanishing for very small differences in positions or momenta between particles under consideration. The reason is that our space-time structure of the process can have in $D = 3$ a kind of “holes”, i.e., regions in which the number of produced particles is very small. This is perhaps the most characteristic observation for fractal (i.e., cascade) processes of the type considered here.

5 Summary and conclusions

In this work we have addressed the problem of the possible space-time fractal structure of the hadronic production process. It is complementary to the possible (multi) fractality leading to fluctuations in the multiparticle distributions and to the possible fractality claimed to exist already on the level of hadronic structure [18]. Although there is a vast literature concerning the possible (multi)fractality in momentum space [19] its space-time aspects are not yet fully recognized with [3]-[5] remaining so far the only representative investigations in this field. Our aim was to extend this investigation a bit further by essentially repeating ideas proposed in [3]-[5] in a numerical form that allowed us to check in more detail the conjectures made there, showing the limits of their applicability.

Our simple model possesses all features called for in [3] - [5]: it shows both intermittency in the phase space (demonstrated in the limiting case of one-dimensional cascade explicitly in Fig. 4) and (approximate) power law distribution of the production points in the space-time (cf. Fig. 5). As we have more constraints on the phase space behaviour imposed by the, for example, expected energy dependence of the multiplicity $\langle N \rangle$, we have not much freedom in choosing decay parameters k . Our distribution $P(k)$ is surely not the only possible one. However, whatever shape we choose for $P(k)$ it should reproduce the expected energy dependence of multiplicity, $\langle N(M) \rangle \sim M^{0.4 \div 0.5}$. Therefore the only really free parameters in our study were the time evolution parameter τ and dimensionality of the cascade, D . Here only two extreme values of $D = 1$ and $D = 3$ were studied and two also, in a

sense, extreme behaviours of $\tau = \text{const}$ and $\tau \sim \frac{1}{M}$ were used. All three: M , D and τ were found to influence the $C_2(Q)$ observable characterising BEC, cf. Fig. 6.

We therefore conclude that BEC are, indeed, substantially influenced by the fact that our process is of the cascade type as was anticipated in [3]-[5], although probably not to the extent expected there (which, however, has not been quantified there). However, in practical applications, i.e., in the eventual fitting of experimental data, there are many points which need further clarification. The most important is the fact that data are usually collected for a range of masses M and among directly produced particles are also resonances. Therefore, one has first to specify the form of the distribution $P(\frac{M}{\mu})$ which will influence to some extent our results. In particular changes in μ due to the production from resonances will shorten our cascade considerably. The possible effect can be to some extent deduced from our results by comparing data with $M = 100$ GeV with those with $M = 40$ GeV and $M = 10$ GeV. The BEC will be effective only in conjunction with precise studies of distributions in the phase-space (like $P(\frac{M}{\mu})$), intermittency and momentum and rapidity distributions. These studies should to some extent fix the distribution of decay parameters $P(k_{i,j})$. Only then one can fit data to different parameters τ characterizing the space-time structure of the source.

One should realize at this point that there were already attempts to study the BEC with power-like (Lorentzian type) shape of source function: $\rho(\xi) = \frac{3}{4\pi^2 R^4} \frac{1}{(1+\xi^2/R^2)^{5/2}}$ which leads to $C_2 = 1 + \exp(-2RQ)$ (with $\xi^2 = (x^2 + y^2 + z^2) + (ct)^2$ and Q defined as in our case) [20]. Such a form is nearest to our $\rho(r)$ and, as it turns out, gives the best fit (in terms of the χ^2 -values) to data considered in [20]. However, differences between this fit and other more conventional ones (i.e., based on gaussian or exponential shapes of the source) were not dramatic. This means, that in reality it will be very difficult to establish by means of BEC the possible existence of fractal structure of the emitting source. Perhaps the event-by-event analysis of data with some preselection of the initial conditions (in terms of energy, centrality, multiplicity etc.) will be necessary in order to perform such investigations.

Our approach must be regarded as preliminary because of our choice of treatment of BEC. Notwithstanding its obvious deficiencies (already mentioned in [7]) it seems, however, fully adequate for the present study which is, as mentioned above, of only limited scope. However, even in such form it seems to indicate the relevance of the fact of a possible fractal structure of the space-time of the emitting region preventing

particles from different branches to be in the same emitting cell irrespectively of the smallness of differences in their positions or momenta. This stresses the problem of the distance in the cascade and the like, recently discussed in [21].

Figure Captions:

Fig. 1 The scheme of our cascade process.

Fig. 2 Example of rapidity distributions of secondaries for totally symmetric (a) and totally asymmetric one-dimensional cascades with asymmetric (b) and symmetric (c) emission of particles calculated for $M = 40$ GeV, $\mu = 0.3$. Histograms are for fixed k : $P(k) = \delta(k - 0.25)$. Open symbols display results for cascades with $k_{1,2}$ distributed randomly according to $P(k) = (1 - k)^a$ with $a = 1$ (in both cases multiplicities are the same: $N_{(a)} \simeq 11.5$ and $N_{(b,c)} \simeq 4.5$). Full lines present the most probable (one dimensional) thermal-like distributions given by (4) with $\beta = 0.028$ for (a) and $\beta = -0.133$ for (b) and (c), respectively, (calculated as in [13] for $N_{(a)} = 11.5$ and $N_{(b,c)} = 4.5$).

Fig. 3 Multiplicity distributions $P(N)$ for (one dimensional) cascades of masses $M = 10, 40, 100$ GeV (for $\mu = 0.3$ GeV and $k_{1,2}$ given by the same triangle distributions $P(k)$ as in Fig. 2): (a) - symmetric case with respective mean values $\langle N \rangle = 7.39, 14.67, 22.76$ and dispersions $\sigma = 2.37, 5.12, 8.47$; (b) - asymmetric case with $\langle N \rangle = 3.97, 4.88, 5.48$ and $\sigma = 1.07, 1.27, 1.41$.

Fig. 4 Example of 2^{nd} and 3^{th} scaled “horizontal” moments F_l as function of the number of bins $n_{bin} = Y/\delta y$ (where Y is the rapidity range considered and δy the bin size) for one-dimensional cascade of $M = 40$ GeV and $\mu = 0.3$ GeV with $k_1 = k_2 = 0.25$ and $k_{1,2}$ chosen randomly in the same way as in Fig. 3. The results for $M = 100$ GeV are essentially identical.

Fig. 5 Density distribution of the production points $\rho(r)$ for one-dimensional cascades ($r = \sqrt{x^2}$, $\mu = 0.3$ GeV) - left panels, and for three-dimensional cascades ($r = \sqrt{x^2 + y^2 + z^2}$, $\mu = 0.14$ GeV) - right panels. Two different choices of the evolution parameter τ are considered: $\tau = 0.2$ fm - upper panels, and $\tau = 0.2/M$ (in fm, the mass M is the parent mass in a given vertex) - lower panels. Each panel shows results for three different masses M of the source: $M = 10, 40$ and 100 GeV. In all cases k is chosen from the same triangle distribution as in Fig. 2.

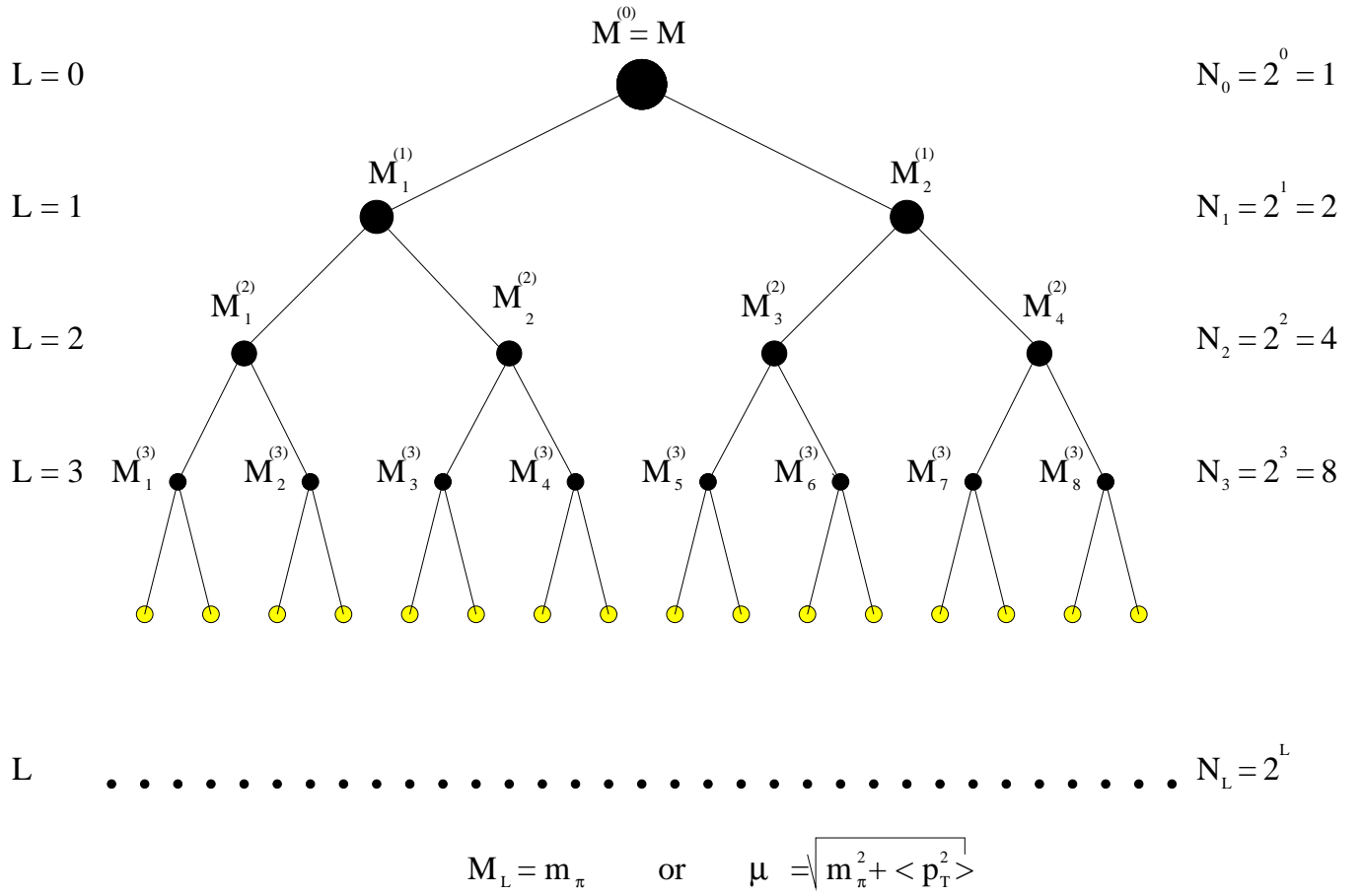
Fig. 6 The $C_2(Q = |p_i - p_j|)$ for the sources presented in Fig. 5: left panels - one-dimensional cascades, right panels - three-dimensional cascades; upper panels - time evolution parameter is set constant and equal $\tau = 0.2$ fm, lower panels - time evolution parameter is chosen as $\tau = 0.2/M$ (in fm, the mass M is the

parent mass in a given vertex). Each panel shows results for three different masses M of the source: $M = 10, 40$ and 100 GeV. In all cases k is chosen from the same triangle distribution as in Fig. 2.

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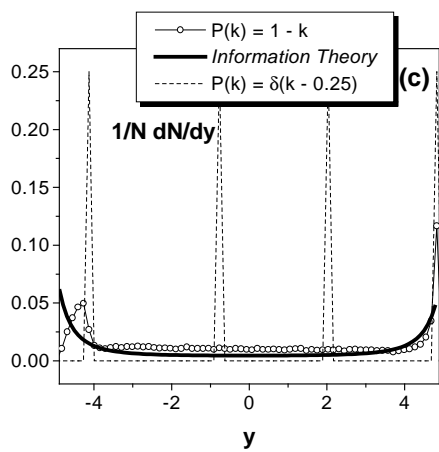
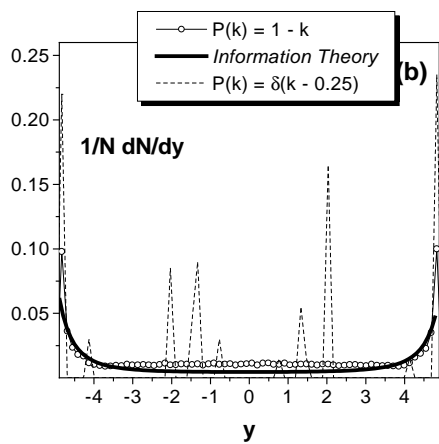
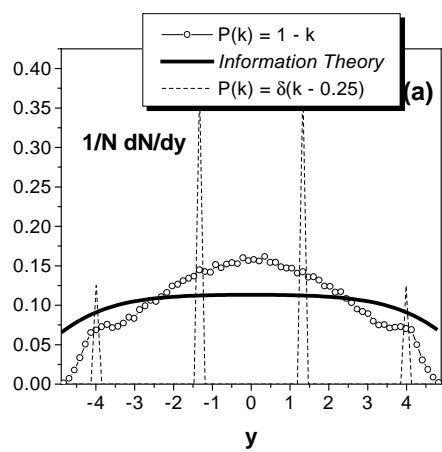


Fig.2

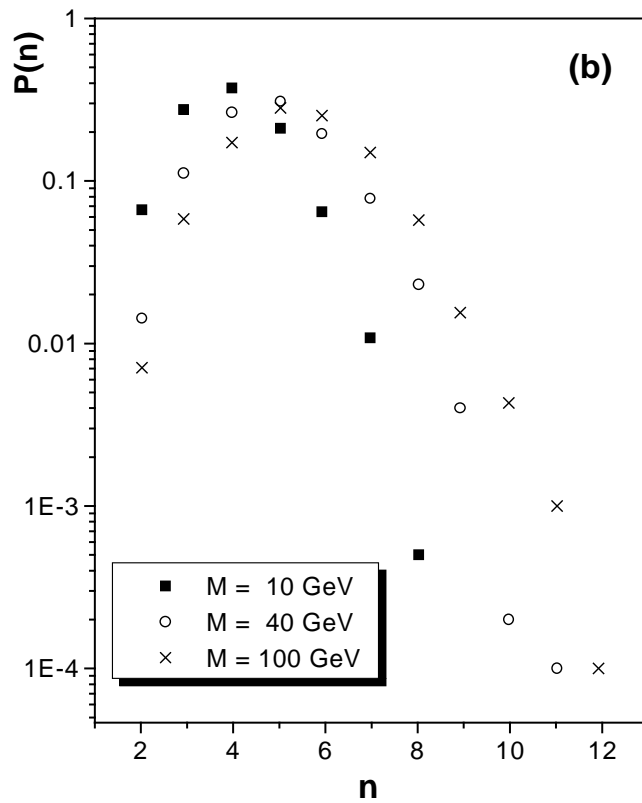
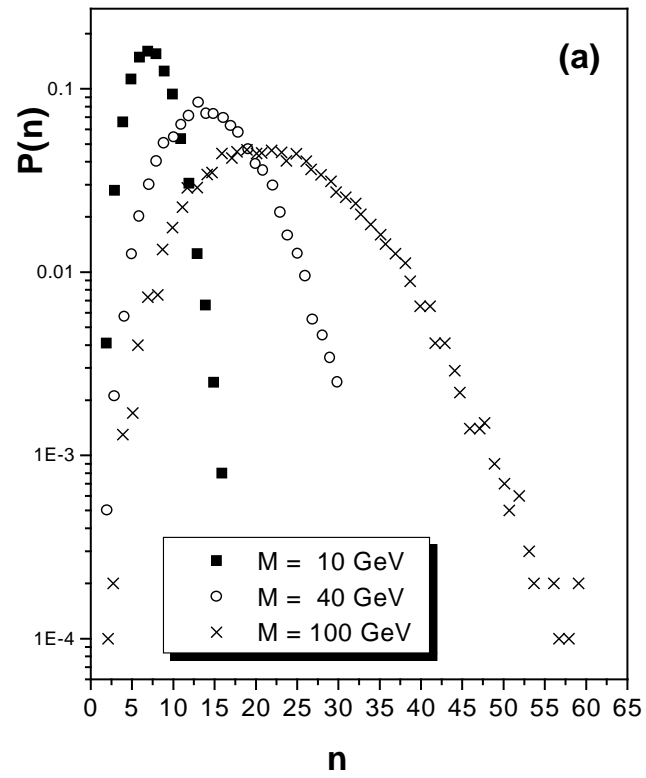


Fig.3

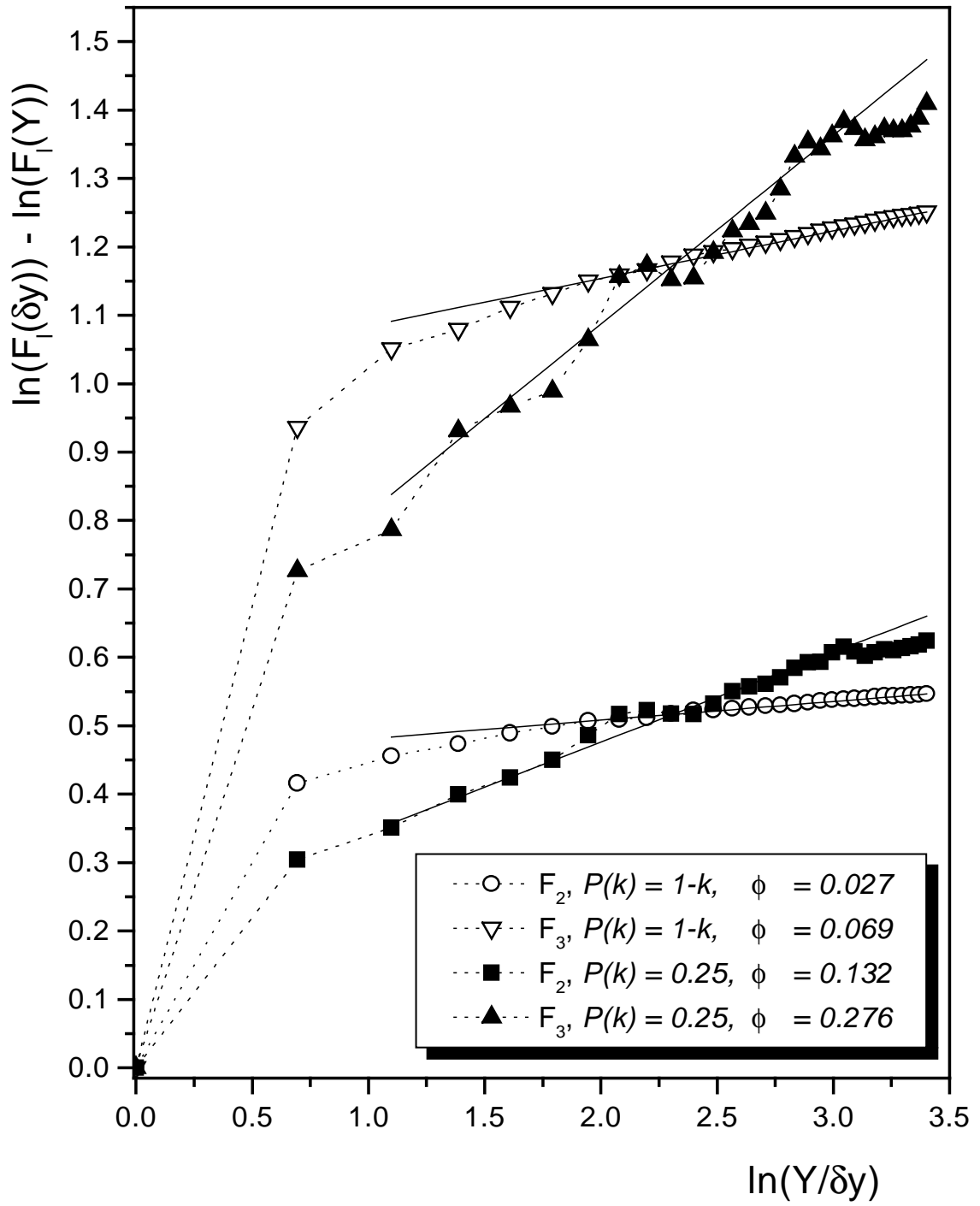


Fig.4

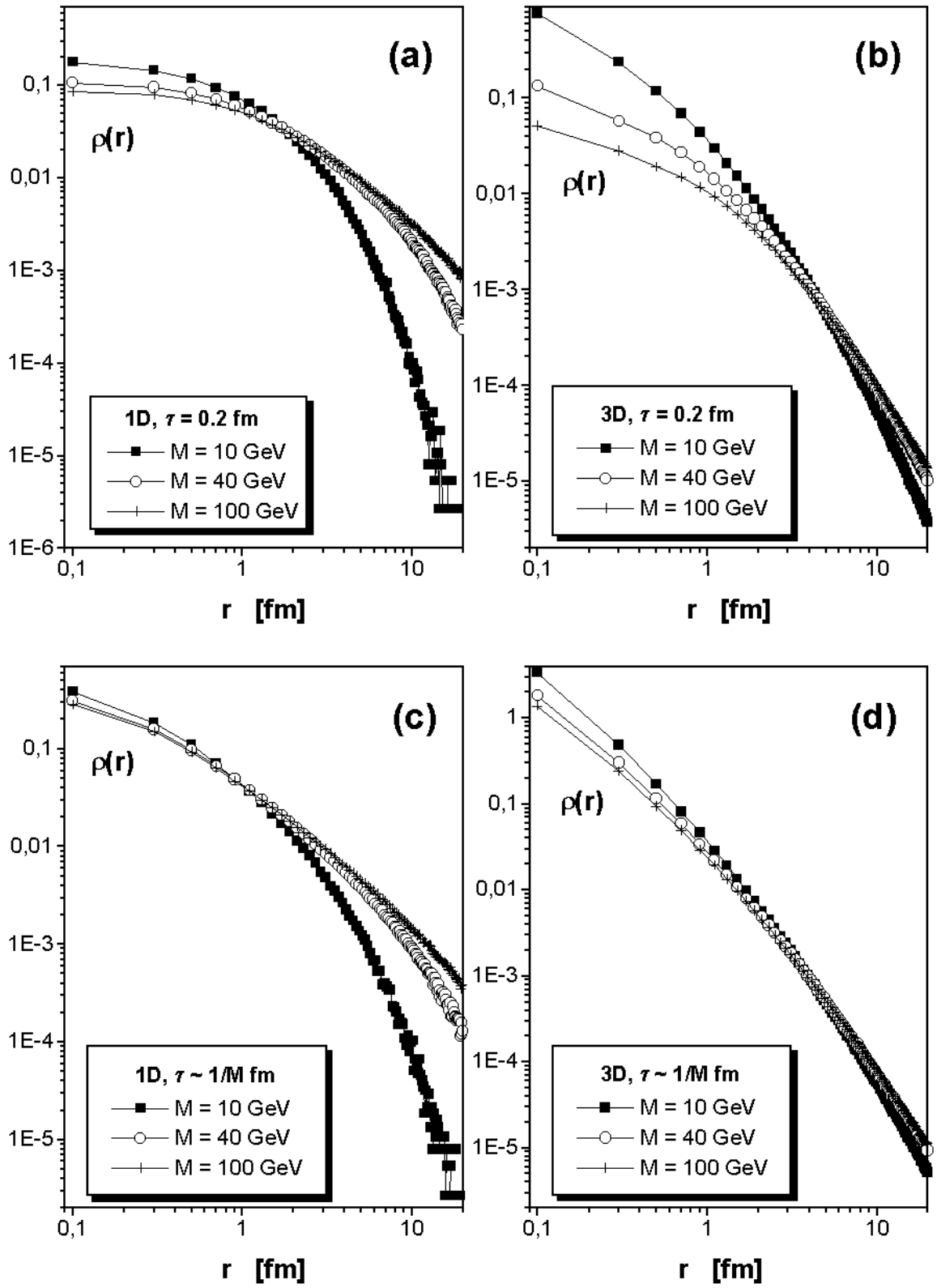


Fig.5

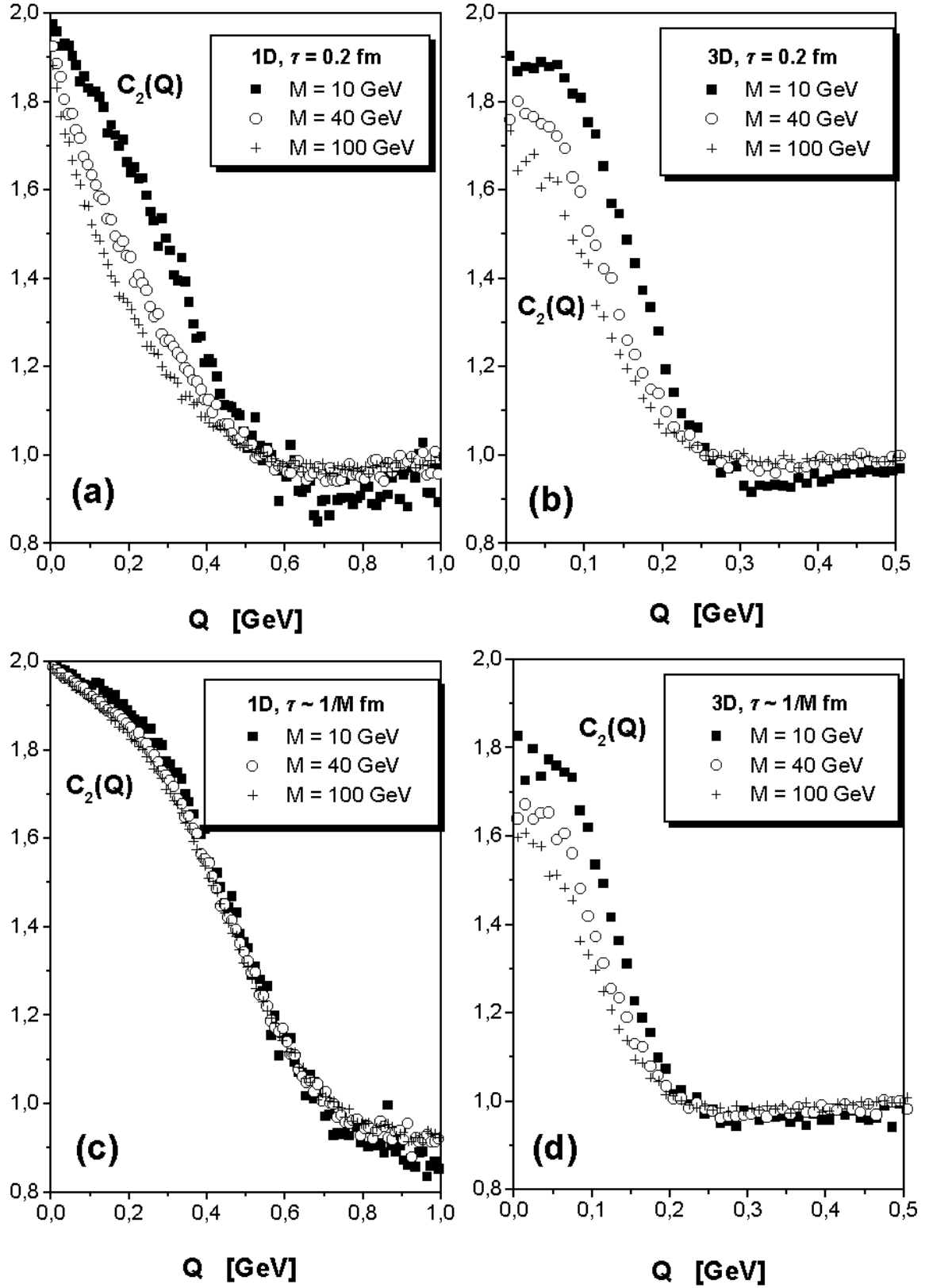


Fig.6